The Informational Structure of Einselection: Observational Entropy in Interaction-Free Measurements

Michael Frank Martin
Saratoga, California
`mfmartin@gmail.com`

December 7, 2025

Abstract

The transition from quantum superpositions to objective classical reality is characterized by the selection of a preferred basis (einselection) and the proliferation of information (Quantum Darwinism). Zurek's "predictability sieve" identifies pointer states through a dynamical criterion: states that survive are those the environment fails to scramble. We demonstrate that this dynamical selection corresponds to an information-theoretic criterion: the minimization of Observational Entropy (OE). While decoherence provides the mechanism for basis selection, OE provides the accounting. The pointer basis emerges as the unique coarse-graining that balances the observer's information gain against the system's coherence loss—a trade-off we term the *informational saddle point*. We test this correspondence by analyzing the High-Efficiency Quantum Interrogation experiment of Kwiat, Zeilinger, et al. (1999). Their Figure 3 reveals an efficiency peak at finite N, where N is the number of interrogation cycles. We show that this peak coincides with the minimum of Observational Entropy, providing empirical evidence that dynamical stability (the sieve) and informational efficiency (OE minimization) identify the same classical limit.

1 Introduction

The emergence of a definite, objective classical world from the underlying quantum substrate remains the central puzzle of quantum foundations. The theory of decoherence explains how interactions between a quantum system ($\mathcal{S}$) and its environment ($\mathcal{E}$) suppress off-diagonal terms in the density matrix, selecting a "pointer basis" of robust states [1]. Quantum Darwinism (QD) extends this by positing that objectivity arises because information about these pointer states is redundantly imprinted onto the environment [2].

While decoherence describes the *mechanism* of this transition, we demonstrate that an information-theoretic measure characterizes the resulting classical limit. Specifically, we utilize the framework of Observational Entropy (OE) [3, 4], which measures the uncertainty of a system given a specific coarse-grained measurement.

Our central claim is that Zurek's dynamical criterion and an information-theoretic criterion identify the same pointer basis. The predictability sieve selects states that resist environmental scrambling; we argue these are precisely the states that minimize Observational Entropy. This correspondence is not accidental. The environment acts as an information channel, continuously querying the system. A basis that minimizes OE is one where the channel extracts maximal information with minimal back-action—exactly the condition for dynamical stability.

This minimization reflects a fundamental trade-off. A measurement that couples too strongly to the system (high resolution) destroys coherence through back-action, increasing entropy. A measurement that couples too weakly (low resolution) allows the system to entangle with unmonitored environmental degrees of freedom, also increasing entropy. The pointer basis occupies the saddle point between these failure modes.

In this paper, we test this correspondence using the "Interaction-Free Measurement" experiment of Kwiat, Zeilinger, et al. [5]. This experiment provides an ideal model system because its efficiency depends explicitly on the measurement strength—parametrized by the number of interrogation cycles N—allowing us to trace the trade-off quantitatively.

2 Theoretical Framework

2.1 Observational Entropy

Observational Entropy S_{obs} generalizes Boltzmann entropy to quantum systems subject to coarse-grained measurements [3]. Given a density matrix ρ and a coarse-graining defined by orthogonal projectors $\{P_i\}$ (where $\sum P_i = I$), S_{obs} is defined as:

$$S_{obs}(\rho, \{P_i\}) = -\sum_i p_i \ln p_i + \sum_i p_i \ln V_i \tag{1}$$

where $p_i = \mathrm{Tr}(\rho P_i)$ is the probability of the macrostate, and $V_i = \mathrm{Tr}(P_i)$ is the volume of the macrostate subspace. The first term is the Shannon entropy of the macroscopic distribution; the second term is the Boltzmann entropy (internal uncertainty) of the macrostates.

3 Case Study: The Kwiat-Zeilinger Experiment

We analyze the "High-Efficiency Quantum Interrogation" experiment [5], which utilizes the Quantum Zeno Effect to detect an absorbing object (the "Bomb") without triggering it.

3.1 The Setup and the "Query"

The experiment uses a photon cycling N times through a loop. In each cycle:

1. A polarization rotator (Quarter Wave Plate - QWP) rotates the polarization by a small angle $\Delta\theta = \pi/2N$.
2. The photon passes through a path containing the object.
3. If the object is present, it acts as a "Zeno" measurer, projecting the photon back to the Horizontal (H) state.
4. If the object is absent, the rotation accumulates, resulting in a Vertical (V) state after N cycles.

From an information-theoretic perspective, the rotation $\Delta\theta$ encodes the "query." A large rotation (corresponding to a small N) asks "Is the bomb there?" loudly, incurring a high probability of explosion (a high quantum cost). A small rotation (large N) asks in a "whisper," reducing explosion risk but requiring the system to maintain coherence for a longer duration (a classical/environmental cost).

3.2 Defining the Macrostates

To apply Observational Entropy, we define the three terminal macrostates of the experiment:

1. **Safe Detection ($\mathcal{M}_{Safe}$):** The photon exits the loop in a defined polarization state (V or H). Information is gained; the system is preserved.
2. **Explosion ($\mathcal{M}_{Exp}$):** The photon is absorbed by the object. The system is destroyed; energy is thermalized.
3. **Loss ($\mathcal{M}_{Loss}$):** The photon is scattered by the optical components (mirrors/lenses) before the cycle completes. The signal is lost to the environment.

3.3 Minimization of Observational Entropy

Kwiat et al. [5] define efficiency η as the probability of detecting the object without absorbing the photon. Figure 3 of their paper reveals a critical feature: **efficiency peaks at a finite N and then degrades.**

We map this directly to Observational Entropy. The uncertainty S_{obs} is minimized when the system successfully sorts the photon into the safe "Detection" macrostates (V or H). Using the recursive probability relations, we derive the exact probabilities for an interrogation of N cycles with reflectivity R:

$$P(\mathcal{M}_{Safe}) = \left[R\cos^2\left(\frac{\pi}{2N}\right)\right]^N \tag{2}$$

$$P(\mathcal{M}_{Exp}) = (1 - P(\mathcal{M}_{Safe}))\frac{\sin^2(\frac{\pi}{2N})}{1 - R\cos^2(\frac{\pi}{2N})} \tag{3}$$

$$P(\mathcal{M}_{Loss}) = (1 - P(\mathcal{M}_{Safe}))\frac{(1-R)\cos^2(\frac{\pi}{2N})}{1 - R\cos^2(\frac{\pi}{2N})} \tag{4}$$

The Observational Entropy is given by:

$$S_{obs} = -\sum_i P(\mathcal{M}_i)\ln P(\mathcal{M}_i) + \sum_i P(\mathcal{M}_i)\ln V_i \tag{5}$$

where V_i represents the phase-space volume of the macrostate. The "Safe" state is a pure quantum state (low volume, $V_{Safe} \approx 1$), while "Explosion" and "Loss" correspond to thermalization of the photon energy into the macroscopic environment (high volume, $V_{Dest} \gg 1$). We assign these phase-space volumes based on the thermodynamic distinction between coherent and thermalized states. The "Safe" outcomes correspond to a single optical mode ($V_{Safe} \approx 1$). In contrast, the "Explosion" and "Loss" outcomes involve the absorption of the photon's energy by a macroscopic reservoir at temperature T. The resulting increase in thermodynamic entropy, $\Delta S \approx \hbar\omega/k_B T$, implies an exponential growth in the effective phase-space volume ($V_{Dest} \sim e^{\Delta S} \gg 1$). Crucially, our results are robust to the precise magnitude of this volume: because the entropy production is dominated by the linear term $(1 - P_{Safe})\ln V_{Dest}$, the location of the entropy minimum N_{opt} coincides with the efficiency peak regardless of the specific value of V_{Dest}, provided the hierarchy $V_{Dest} \gg V_{Safe}$ holds. For the numerical visualization in Figure 1, we adopt a conservative effective volume $\ln(V_{Dest}) = 5.0$ to render the trade-off visible on a shared axis.

3.3.1 Interpretation of Figure 3

Our numerical analysis (see Figure 1) confirms that the minimum of S_{obs} coincides with the maximum of $P(\mathcal{M}_{Safe})$.

- **Left of Peak (Small N):** The rotation $\Delta\theta$ is too large. The "measurement" is too invasive, causing the Zeno effect to fail. Here, S_{obs} is high because the "Explosion" outcome is probable, generating entropy via photon thermalization.
- **Right of Peak (Large N):** The rotation $\Delta\theta$ is too small; the query is too subtle. The system must persist for too long, allowing environmental degrees of freedom (mirror losses) to entangle with the photon. Consequently, S_{obs} increases because the signal vanishes into the noise of $\mathcal{M}_{Loss}$.

Crucially, while the experimental efficiency η only tracks the survival of the photon, Observational Entropy accounts for the total information state of the universe (including the scattered photons in $\mathcal{M}_{Loss}$). The correspondence between the peak η and minimum S_{obs} demonstrates that the "best" experimental result is not arbitrary; it is physically located at the point where the system maintains maximum distinctness from the environment.

4 Discussion: The Cost of Information

4.1 The Entropic Trade-off: Uncertainty vs. Dissipation

The Kwiat-Zeilinger protocol illustrates a general tension in quantum measurement: the conflict between **resolution** and **isolation**. The rotation angle $\Delta\theta = \pi/2N$ encodes the "query strength."

The Quantum Constraint (Low N): A large rotation asks "Is the bomb there?" loudly. This corresponds to a strong measurement with high back-action. In the language of the Zeno effect, the measurement is too invasive, projecting the system out of the coherent subspace before the protocol can complete. This is analogous to the Heisenberg limit in position measurement: high spatial resolution imparts high momentum disturbance, destroying the state's future evolution.

The Thermodynamic Constraint (High N): A small rotation asks in a "whisper," minimizing back-action. However, this requires the system to remain in the apparatus for a longer duration. Since the apparatus is an open quantum system (mirror reflectivity $R < 1$), extending the duration increases the entanglement with environmental degrees of freedom (phonons in the mirrors). Here, the limit is not Heisenberg uncertainty, but **thermodynamic dissipation**.

The pointer basis (the efficiency peak at $N_{opt} \approx 16$) emerges at the saddle point between these two distinct failure modes: **quantum back-action** (dominant at low N) and **environmental decoherence** (dominant at high N).

4.2 Correspondence with the Predictability Sieve

Zurek's predictability sieve identifies pointer states by minimizing entropy production over initial states [1]. It asks: which states resist environmental scrambling? The answer depends on the interaction Hamiltonian H_{int}; states that commute with H_{int} (or nearly so) survive, while superpositions decohere.

We propose that OE minimization provides the information-theoretic *interpretation* of this dynamical result. The interaction Hamiltonian defines an information channel between system and environment. The environment "queries" the system, extracting information about certain observables. This channel has a preferred basis: the eigenstates of the observable being monitored.

When the system occupies a pointer state, the channel operates cleanly. The environment gains information, but this information is redundant with what an observer could obtain directly—no additional entropy is generated. When the system occupies a superposition, the channel becomes noisy. The environment's query projects the system stochastically, generating entropy through the randomness of the outcome.

The Kwiat-Zeilinger experiment makes this trade-off explicit:

- The bomb (when present) acts as a channel, querying "is the photon in the V-path?" at each cycle.
- Small N corresponds to a high-bandwidth channel: each query is loud, the answer is definite, but back-action (absorption probability) is severe.
- Large N corresponds to a low-bandwidth channel: each query is gentle, but the photon must traverse more optical elements, leaking information to unmonitored modes (mirror losses).
- The efficiency peak at N_{opt} represents the channel capacity maximum—the point where information transfer to the observer is maximized relative to information loss to the environment.

This mapping also suggests potential follow-ups. The OE framework applies across different decoherence channels. Decoherence models often focus on **phase damping** (pure dephasing), where off-diagonal terms vanish while energy is conserved. The Kwiat experiment, by contrast, models an **amplitude damping** channel (photon loss), where the system leaks energy into the bath. The fact that OE minimization correctly identifies the robust states in this dissipative regime suggests that the principle is general: whether the environment scrambles phases or absorbs energy, the pointer basis remains the configuration that minimizes the joint observer-system entropy. We conjecture that this correspondence extends to other decoherence channels, though demonstration in phase-damping regimes remains future work.

4.3 Information Destruction

The presence of the object destroys the superposition that would otherwise evolve. In the context of Observational Entropy, the "Bomb" acts as a harsh coarse-graining agent. It forces the universe to decide between "Photon here" and "Photon there" at every cycle. The efficiency peak represents the regime where this forced decision yields the highest mutual information between the Observer and the Bomb, minimizing the entropy generated by the "destruction" of the alternative paths.

5 Conceptual Challenges and Clarifications

Having demonstrated a relationship between thermodynamic and dynamical frameworks, it is tempting to speculate about whether OE might offer additional insights. But the usual conceptual challenges remain.

5.1 The Circularity Problem

The question of circularity remains here: does defining the coarse-graining presuppose the classical structure we wish to derive? As noted by Kastner [6], criteria that rely on pre-existing partitions or coarse-grainings may be tautological. The connection between OE and QD demonstrated here does not resolve this tension and does not need to. Since both the Predictability Sieve and OE depend on an initial system-environment partition, showing they map to the same result demonstrates the

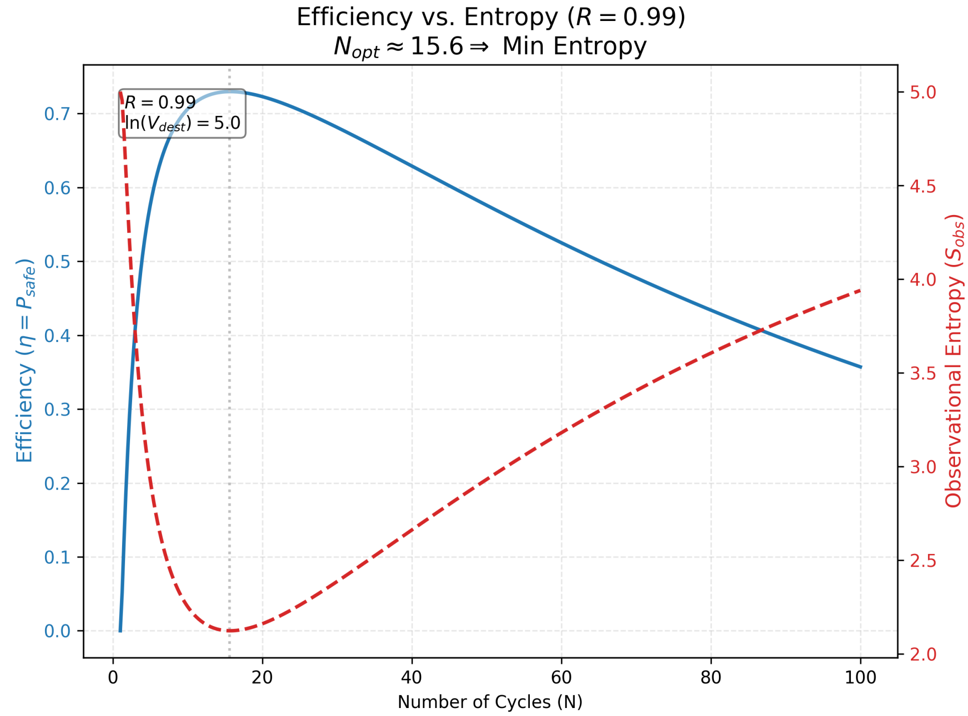

Figure 1: Efficiency of the interaction-free measurement as a function of the number of cycles N. The solid line represents the theoretical efficiency $\eta = P(\mathcal{M}_{Safe})$ derived from the model in [5] (specifically Endnote 12), with reflectivity $R \approx 0.99$. The peak efficiency corresponds to the minimization of Observational Entropy, balancing the quantum risk of triggering the bomb (dominant at low N) against environmental decoherence/loss (dominant at high N). Adapted from Figure 3 of Kwiat et al. [5].

consistency of the thermodynamic and dynamical frameworks, regardless of the ultimate origin of the partition.

5.2 The Measurement Problem and Improper Mixtures

We want to be precise about what this framework does and does not explain.

Decoherence transforms a pure-state superposition into an improper mixture—a reduced density matrix diagonal in the pointer basis. But an improper mixture is not a proper mixture; it does not represent ignorance about a pre-existing definite state. The "measurement problem" asks how (or whether) the improper mixture becomes proper—how one outcome is selected from the diagonal.

OE minimization does not answer this question. It does not explain collapse, nor does it select a single outcome from the ensemble. What it explains is different: why the improper mixture *looks classical*.

OE minimization explains the *texture* of the decohered state—why it resembles a classical probability distribution rather than quantum noise. It does not (and if we follow Kastner [6], cannot) explain the *selection* of a particular outcome from that distribution.

6 Conclusion

We have argued that Zurek's predictability sieve and Observational Entropy minimization identify the same pointer basis—the former through dynamics, the latter through information theory. This correspondence is not coincidental. The environment acts as an information channel; the pointer basis is the channel's preferred encoding. States that survive decoherence are states that communicate efficiently across this channel, maximizing information transfer to observers while minimizing leakage to unmonitored degrees of freedom.

The Kwiat-Zeilinger experiment provides empirical support for this correspondence. Their efficiency peak at finite N represents the informational saddle point: too few cycles and quantum back-action dominates; too many and environmental decoherence dominates. Classical reality—the definite fact of the bomb's presence—emerges not at either extreme but at the balance point where Observational Entropy is minimized.

This framework does not replace decoherence theory or solve the measurement problem. It provides a complementary language—a Rosetta Stone between dynamical and informational descriptions of the quantum-classical boundary. The pointer basis is simultaneously the most stable (Zurek) and the most informative (OE). That these criteria coincide suggests something deep about the relationship between dynamics and information, a connection that warrants further investigation.

References

[1] Wojciech H. Zurek. Decoherence, einselection, and the quantum origins of the classical. *Rev. Mod. Phys.*, 75(3):715–775, 2003.

[2] Wojciech H. Zurek. Quantum Darwinism. *Nature Physics*, 5(3):181–188, 2009.

[3] Dominik Šafránek, J. M. Deutsch, and Anthony Aguirre. Quantum coarse-grained entropy and thermalization in closed systems. *Phys. Rev. A*, 99(1):012103, 2019.

[4] Dominik Šafránek, Anthony Aguirre, Joseph Schindler, and J. M. Deutsch. A brief introduction to observational entropy. *arXiv preprint arXiv:2008.04409*, 2020.

[5] P. G. Kwiat, A. G. White, J. R. Mitchell, O. Nairz, G. Weihs, H. Weinfurter, and A. Zeilinger. High-efficiency quantum interrogation measurements via the quantum Zeno effect. *Phys. Rev. Lett.*, 83:4725, 1999.

[6] R. E. Kastner. 'Einselection' of pointer observables: The new H-theorem? *Stud. Hist. Phil. Mod. Phys.*, 48:56–58, 2014.