Bit from It: Observational Entropy in Interaction-Free Measurements

Michael Frank Martin
Saratoga, California
mfmartin@gmail.com

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Abstract

The quantum Zeno interrogation experiment of Kwiat *et al.* (1999) demonstrates that information about the presence of an absorbing object can be obtained with arbitrarily high efficiency — approaching unity in the lossless limit — via repeated weak measurements that inhibit the photon's coherent evolution. I analyze this experiment using the observational entropy framework of Šafránek, Aguirre, and Deutsch, which provides a unified treatment of information-theoretic and thermodynamic entropy through coarse-grained measurements. The framework reveals how the observer's uncertainty about the measurement outcome, quantified by observational entropy S_{obs}, decreases as the number of interrogation cycles N increases. In the limit $N \to \infty$, the efficiency $\eta \to 1$ and $S_{obs} \to 0$: complete certainty about the object's presence is achieved without any photon absorption, illuminating the relational character of quantum information and the role of the observer in extracting knowledge from physical systems.

1 Introduction

The 1999 experiment by Kwiat *et al.* [1] stands as one of the most striking demonstrations of the quantum Zeno effect in an optical setting. By combining the interferometric ideas of Elitzur and Vaidman [2] with repeated polarization measurements, the experiment achieved "quantum interrogation" of an absorbing object with efficiencies exceeding the 50% threshold of the original interaction-free measurement proposal. In the idealized lossless limit, the efficiency η—defined as the fraction of measurements that successfully detect the object without photon absorption—approaches unity as the number of cycles $N \to \infty$.

This remarkable result invites a careful information-theoretic analysis. What exactly happens to the information content of the measurement as N increases? How does the observer's uncertainty evolve? These questions cannot be adequately addressed using the von Neumann entropy alone, since pure states have zero von Neumann entropy regardless of what is known about them from a measurement perspective.

The observational entropy framework developed by Šafránek, Aguirre, Deutsch, and collaborators [3, 4, 5] provides precisely the tools needed for this analysis. Observational entropy S_{obs} quantifies the uncertainty an observer would have about a system's state given a particular coarse-graining—that is, a specification of what measurements the observer can perform. It reduces to the von Neumann entropy when the coarse-graining is maximally fine (measuring the density matrix itself), but in general it exceeds the von Neumann entropy, capturing the additional uncertainty arising from the observer's limited measurement capabilities.

In this paper, I apply the observational entropy framework to the Kwiat *et al.* experiment. The analysis reveals a direct relationship between the interrogation efficiency η and the observational entropy S_{obs}: as $\eta \to 1$, we have $S_{obs} \to 0$. This corresponds to a transition from maximal uncertainty (when the measurement cannot distinguish between outcomes) to complete certainty (when the object's presence is determined with probability one, without absorption). The thermodynamic and information-theoretic aspects of the measurement are unified in a single framework.

2 The Quantum Zeno Interrogation Scheme

2.1 Experimental Setup

The essential idea of the Kwiat *et al.* experiment is illustrated in Figure 1. A photon with initial horizontal polarization $|H\rangle$ is made to circulate N times through an optical system consisting of a polarization rotator and a polarization interferometer. On each cycle, the rotator advances the polarization angle by $\Delta\theta = \pi/2N$. After N cycles in the absence of any object, the photon's polarization has rotated to vertical $|V\rangle$.

The polarization interferometer separates the $|H\rangle$ and $|V\rangle$ components of the photon and recombines them with the same relative phase. If an opaque object is placed in the vertical arm, only the horizontal component is transmitted. At each cycle, either the photon is absorbed by the object (with probability $\sin^2 \Delta\theta$) or it survives and is projected back into the horizontal polarization state (with probability $\cos^2 \Delta\theta$).

This constitutes a quantum Zeno effect: the repeated "measurements" by the object—or more precisely, the possibility of absorption that projects the state—inhibit the coherent rotation of polarization. After N cycles, if the photon has not been absorbed, its polarization remains horizontal, unambiguously indicating the presence of the object.

2.2 Efficiency and the Lossless Limit

Following Kwiat *et al.* [1], we define the efficiency as

$$\eta = \frac{P_{QI}}{P_{QI} + Pabs}, \tag{1}$$

where P_{QI} is the probability that the photon is detected without absorption (a successful quantum interrogation) and P_{abs} is the probability that the photon is absorbed by the object.

For the idealized lossless system, the analysis proceeds straightforwardly. The probability of surviving all N cycles without absorption is

$$P_{QI} = \cos^{2N}\left(\frac{\pi}{2N}\right), \tag{2}$$

and the complementary probability of absorption at some cycle is

$$P_{abs} = 1 - \cos^{2N}\left(\frac{\pi}{2N}\right). \tag{3}$$

For large N, we can expand $\cos(\pi/2N) \approx 1 - \pi^2/(8N^2)$, giving

$$P_{QI} \approx 1 - \frac{\pi^2}{4N}, \quad P_{abs} \approx \frac{\pi^2}{4N}. \tag{4}$$

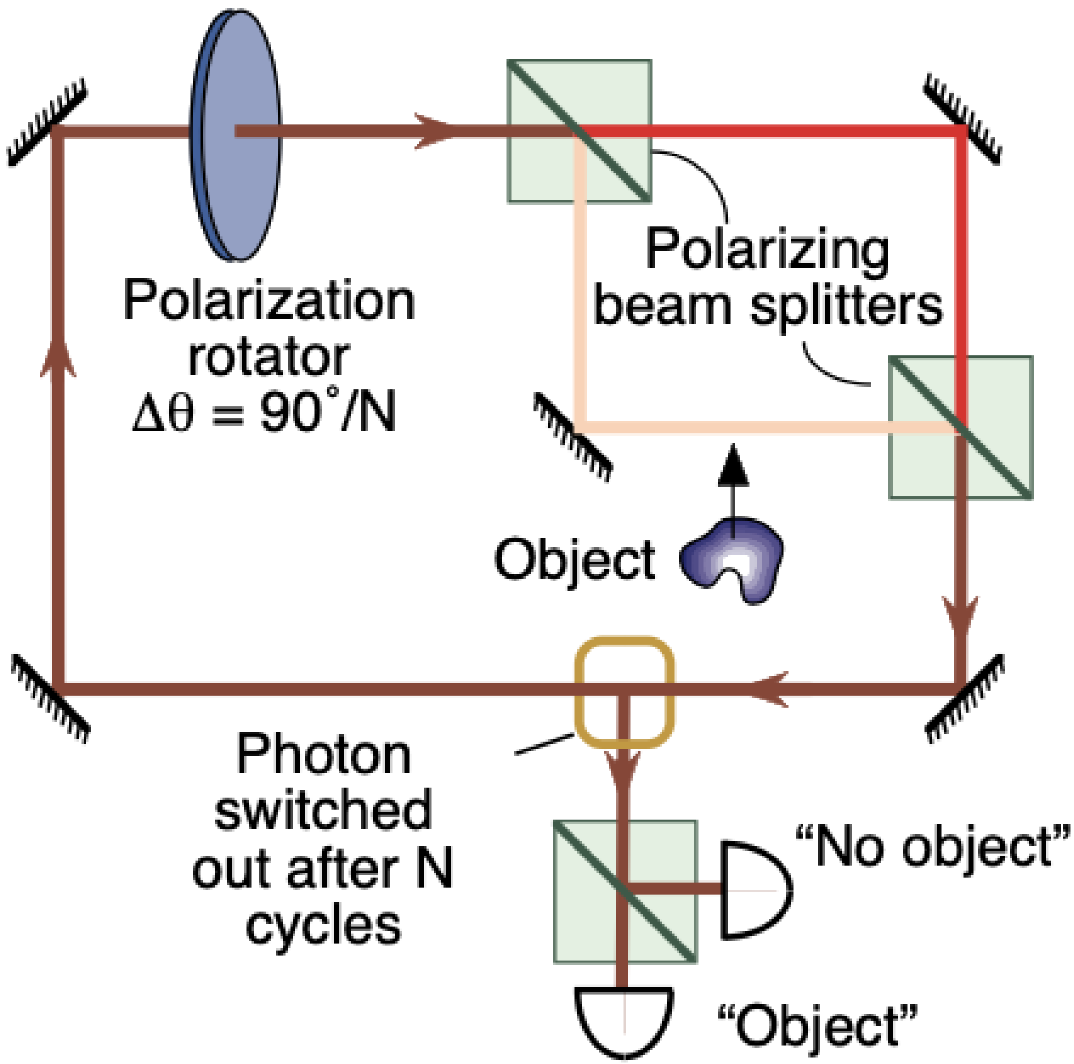

Figure 1: Schematic of the high-efficiency quantum interrogation scheme (adapted from Kwiat *et al.* [1]). With no object present, the photon's polarization rotates stepwise from horizontal to vertical over N cycles. An object in the V-polarized arm inhibits this evolution via the quantum Zeno effect, so that the final polarization unambiguously indicates the object's presence or absence.

Thus the efficiency approaches unity as

$$\eta \approx 1 - \frac{\pi^2}{4N} \quad \text{as } N \to \infty. \tag{5}$$

In a real system with optical losses, the situation is more complex. Footnote 12 of Kwiat *et al.* [1] provides the general expressions:

$$P_{QI} = T_{\text{empty}} \cos^{2N}(\Delta\theta)\, T_{\text{rec}}^{N-1}, \tag{6}$$

$$P_{abs} = T_{\text{obj}} \sin^2(\Delta\theta) \frac{1 - (T_{\text{empty}} T_{\text{rec}} \cos^2 \Delta\theta)^N}{1 - T_{\text{empty}} T_{\text{rec}} \cos^2 \Delta\theta}, \tag{7}$$

where T_{empty}, T_{obj}, and T_{rec} are the single-cycle transmission probabilities for the empty arm, the arm with the object, and the recycling path, respectively.

Figure 2 shows the experimental realization, and Figure 3 displays the measured efficiencies for several system configurations. The key observation is that optical loss fundamentally limits the achievable efficiency: a photon contributing to P_{QI} must survive all N cycles, sampling the loss N times, whereas a photon contributing to P_{abs} is absorbed on average before completing all cycles. Nevertheless, efficiencies up to 73% were observed, significantly exceeding the 50% Elitzur-Vaidman threshold.

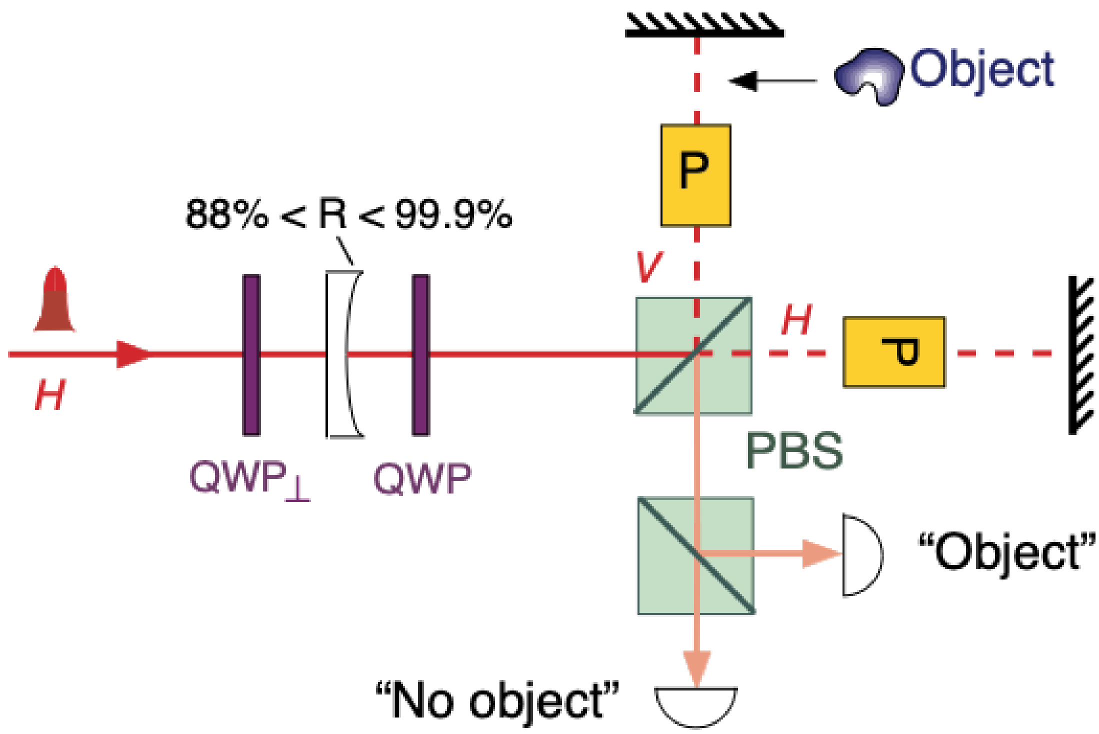

Figure 2: Experimental system for high-efficiency quantum interrogation (from Kwiat *et al.* [1]). A pulsed laser at 670 nm is coupled into a recycling system via a high-reflectivity mirror. Quarter waveplates rotate the polarization, and Pockels cells switch photons out after the desired number of cycles.

3 Observational Entropy Framework

3.1 Definition and Interpretation

The observational entropy framework [3, 4, 5] provides a quantum generalization of Boltzmann entropy that depends explicitly on the coarse-graining available to the observer. Given a density matrix $\hat{\rho}$ and a coarse-graining $\mathcal{C} = \{\hat{P}_i\}$ (a complete set of orthogonal projectors), the observational

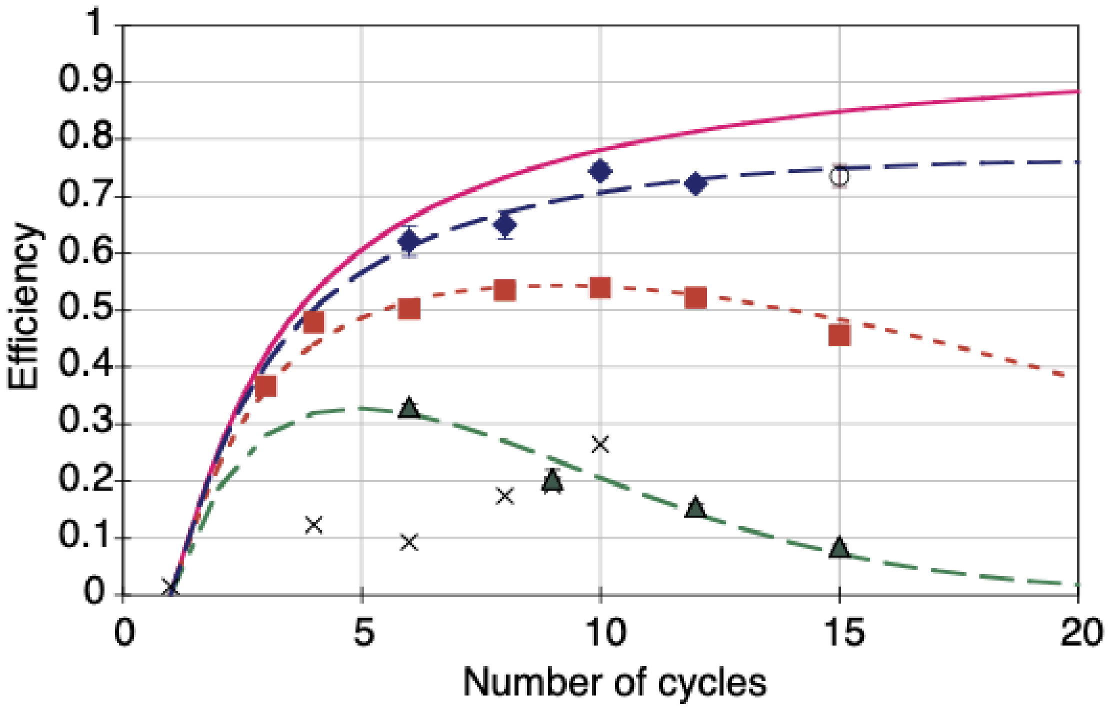

Figure 3: Measured efficiency versus number of cycles for several configurations (from Kwiat *et al.* [1]). The solid curve shows the theoretical prediction for a lossless system. Optical losses cause the efficiency to reach a maximum and then decrease, but efficiencies exceeding the 50% Elitzur-Vaidman limit were achieved.

entropy is defined as

$$S_{obs} \equiv -\sum_i p_i \ln \frac{p_i}{V_i}, \tag{8}$$

where $p_i = \mathrm{Tr}[\hat{P}_i \hat{\rho}]$ is the probability of obtaining outcome i in a measurement, and $V_i = \mathrm{Tr}[\hat{P}_i] = \dim(\mathcal{H}_i)$ is the dimension ("volume") of the corresponding subspace.

This definition can be decomposed into two contributions:

$$S_{obs} = S_{Sh}(p_i) + \langle S_B(i) \rangle_{p_i}, \tag{9}$$

where

$$S_{Sh}(p_i) = -\sum_i p_i \ln p_i \tag{10}$$

is the Shannon entropy of the measurement outcomes, and

$$\langle S_B(i) \rangle_{p_i} = \sum_i p_i \ln V_i \tag{11}$$

is the expected Boltzmann entropy, representing the uncertainty about the microstate within each macrostate.

The observational entropy satisfies fundamental bounds:

$$S_{\mathrm{vN}}(\hat{\rho}) \leq S_{obs}(\hat{\rho}) \leq \ln(\dim \mathcal{H}), \tag{12}$$

with equality on the left when the coarse-graining measures the density matrix itself, and equality on the right when the system appears maximally mixed from the observer's perspective.

3.2 Physical Interpretation

The observational entropy quantifies the uncertainty an observer would infer about the system's initial state by making a measurement, without actually performing it. In the relational interpretation of quantum mechanics [6], this is precisely the relevant quantity: what matters is not some absolute entropy of the quantum state, but the information that an observer can extract through available measurements.

When $S_{obs} = 0$, the observer has complete certainty about the measurement outcome and the system's post-measurement state. When S_{obs} is maximal, the observer's measurement reveals no information—the outcome probabilities are proportional to the macrostate volumes, corresponding to complete ignorance.

4 Application to Quantum Zeno Interrogation

4.1 Coarse-Graining for the Measurement

To apply the observational entropy framework to the Kwiat *et al.* experiment, we must specify the relevant coarse-graining. The measurement has two possible outcomes when an object is present:

1. **Quantum interrogation success (QI)**: The photon survives all N cycles without absorption and is detected with horizontal polarization (after the final Pockels cell rotation), unambiguously indicating the object's presence.
2. **Absorption (abs)**: The photon is absorbed by the object at some cycle $k \in \{1, 2, \ldots, N\}$.

For the coarse-graining, we define two macrostates:

- Macrostate QI has volume $V_{\mathrm{QI}} = 1$, corresponding to a single, definite outcome.
- Macrostate abs has volume $V_{\mathrm{abs}} = N$, since absorption could occur at any of the N cycles. The observer's measurement (final detection or non-detection) does not distinguish *which* cycle caused the absorption.

This assignment reflects the operational reality: a successful quantum interrogation provides complete information about the photon's trajectory (it passed through all N cycles without interacting with the object), while an absorption event leaves uncertainty about when the interaction occurred.

4.2 Entropy Calculation

With this coarse-graining, the observational entropy for the quantum interrogation measurement is

$$S_{obs} = -P_{QI} \ln \frac{P_{QI}}{1} - P_{abs} \ln \frac{P_{abs}}{N}. \tag{13}$$

Expanding this expression:

$$S_{obs} = -P_{QI} \ln P_{QI} - P_{abs} \ln P_{abs} + P_{abs} \ln N. \tag{14}$$

The first two terms constitute the Shannon entropy of the binary outcome distribution:

$$S_{Sh} = -P_{QI} \ln P_{QI} - P_{abs} \ln P_{abs}. \tag{15}$$

The third term is the expected Boltzmann entropy:

$$\langle S_B \rangle = P_{abs} \ln N. \tag{16}$$

4.3 Behavior as a Function of N

Using the asymptotic expressions (4), we can analyze the behavior for large N. Setting $\epsilon \equiv \pi^2/4N \ll 1$, we have $P_{QI} \approx 1 - \epsilon$ and $P_{abs} \approx \epsilon$. The Shannon entropy becomes

$$S_{Sh} \approx -(1-\epsilon)\ln(1-\epsilon) - \epsilon \ln \epsilon \tag{17}$$

$$\approx \epsilon - \epsilon \ln \epsilon + O(\epsilon^2) \tag{18}$$

$$= \frac{\pi^2}{4N}\left(1 - \ln \frac{\pi^2}{4N}\right). \tag{19}$$

The expected Boltzmann entropy is

$$\langle S_B \rangle \approx \epsilon \ln N = \frac{\pi^2}{4N} \ln N. \tag{20}$$

For the total observational entropy:

$$S_{obs} \approx \frac{\pi^2}{4N}\left(1 + \ln \frac{4N^2}{\pi^2}\right) \sim \frac{\pi^2 \ln N}{2N}. \tag{21}$$

This reveals a crucial result: as $N \to \infty$, $S_{obs} \to 0$. The $(\ln N)/N$ decay is faster than $1/N$ but slower than exponential. Physically, this means that increasing the number of cycles not only improves the efficiency η but simultaneously reduces the observer's uncertainty about the measurement outcome to zero.

4.4 Numerical Results

Figure 4 presents a comprehensive view of the efficiency and entropy behavior. Panel (a) shows the efficiency η as a function of N for both the lossless system and a representative lossy configuration. Panel (b) displays the three entropy quantities: observational entropy S_{obs}, Shannon entropy S_{Sh}, and expected Boltzmann entropy $\langle S_B \rangle$.

Several features are noteworthy:

1. The observational entropy S_{obs} initially increases with N for small N, reaching a maximum near $N \approx 3$. This reflects the competition between increasing macrostate volume $V_{\text{abs}} = N$ (which increases $\langle S_B \rangle$) and decreasing absorption probability P_{abs} (which decreases both entropy contributions).

2. For $N \gtrsim 5$, S_{obs} monotonically decreases toward zero as efficiency increases.

3. The Shannon entropy S_{Sh} (uncertainty about which outcome occurs) dominates at small N, while the Boltzmann contribution $\langle S_B \rangle$ (uncertainty about when absorption occurs, given that it does) becomes relatively more important at intermediate N.

4. In the lossy case, the entropies remain bounded away from zero, corresponding to the fundamental limitation on efficiency imposed by optical losses.

Table 1 provides specific numerical values for reference.

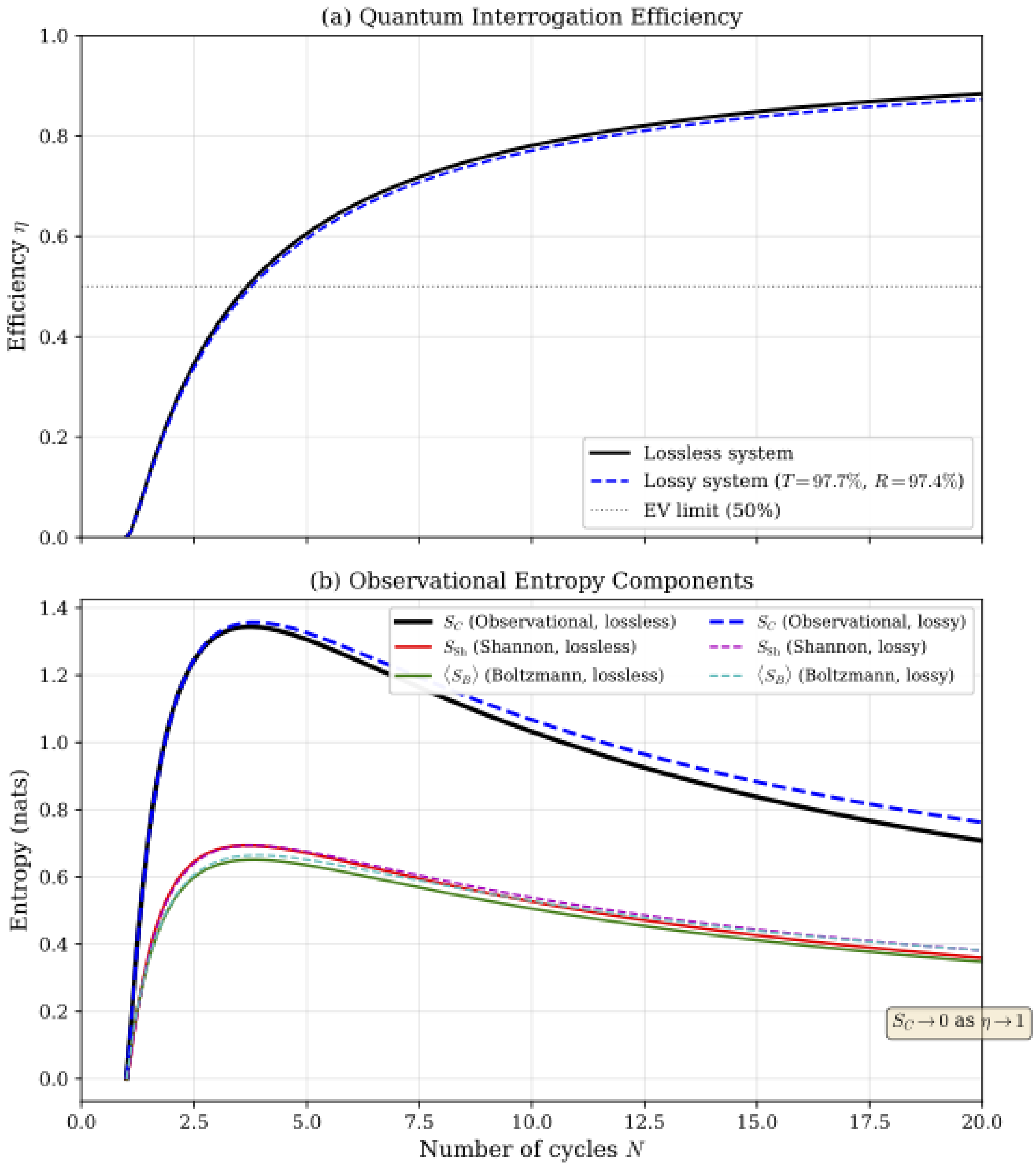

Figure 4: Efficiency and observational entropy analysis of quantum Zeno interrogation. (a) Efficiency η versus number of cycles N, showing the lossless theoretical limit (solid) and a lossy configuration (dashed). (b) Entropy components: total observational entropy S_{obs} (black), Shannon entropy S_{Sh} (red), and expected Boltzmann entropy $\langle S_B \rangle$ (green). As $\eta \to 1$, all entropy measures approach zero, indicating complete certainty about the object's presence without absorption.

Table 1: Efficiency and entropy values for the lossless quantum Zeno interrogation system.

N	η	P_{QI}	P_{abs}	S_{Sh}	$\langle S_B \rangle$	S_{obs}
1	0.000	0.000	1.000	0.000	0.000	0.000
2	0.250	0.250	0.750	0.562	0.520	1.082
5	0.605	0.605	0.395	0.671	0.635	1.306
10	0.781	0.781	0.219	0.526	0.505	1.032
20	0.884	0.884	0.116	0.359	0.348	0.707
50	0.952	0.952	0.048	0.192	0.189	0.381
100	0.976	0.976	0.024	0.111	0.116	0.227

5 Information-Theoretic Interpretation

5.1 The Relational Character of Quantum Information

The observational entropy analysis illuminates a key feature of quantum measurement: information is fundamentally relational. The entropy S_{obs} does not describe an intrinsic property of the photon-plus-object system; rather, it quantifies the uncertainty faced by an observer making a specific type of measurement.

Consider the limit $N \to \infty$. In this case:

- $\eta \to 1$: The photon is never absorbed.
- $S_{obs} \to 0$: The observer gains complete certainty about the object's presence.

The photon never *interacts* with the object in the sense that no energy or momentum is exchanged. Yet information about the object is extracted. How is this possible? The answer lies in the relational structure of the quantum state. The *possibility* of interaction — embodied in the nonzero coupling between photon and object at each cycle — is essential. The Zeno effect works precisely because each cycle could have resulted in absorption but did not. The sequence of non-events carries information.

This is analogous to the famous "negative result" measurements discussed by Renninger [7] and Dicke [8]. The detection of a photon at one location changes the wavefunction elsewhere, even at locations where no physical detector is present. In the Zeno interrogation, the repeated "measurements" by the object (whether or not they result in absorption) continuously update the observer's state of knowledge.

This structure has a classical analogue in the Monty Hall problem [14, 15]. When Monty opens a door to reveal a goat, the contestant gains information not through any direct interaction with the car, but through the elimination of a possibility that *could have* been realized but was not. The host's choice is constrained by his knowledge of where the car is located; the door he does not open carries information precisely because he could have opened it. Similarly, in quantum Zeno interrogation, each cycle where the photon survives without absorption updates the observer's state of knowledge. The object's presence constrains which outcomes are possible; the outcomes that do not occur — the absorptions that could have happened but did not — carry the information. In both cases, the relational structure between observer, system, and the space of possibilities is what enables information transfer without direct causal interaction in the naive sense.

5.2 Entropy and the Second Law

From a thermodynamic perspective, the decreasing observational entropy as N increases might seem puzzling. How can the entropy of a closed system decrease?

The resolution is that S_{obs} is not the thermodynamic entropy of the photon-object system, but the entropy associated with a particular coarse-graining of the observer's measurement. The observer's uncertainty decreases because more cycles provide more opportunities for the Zeno effect to distinguish between the "object present" and "object absent" hypotheses.

The underlying dynamics remain unitary. If we track the full quantum state—including the object, the photon, and all the optical elements—the von Neumann entropy is conserved. What changes is the relationship between the observer and the system, as more information becomes accessible through the measurement.

5.3 Connection to Observational Entropy of Thermodynamic Systems

Šafránek *et al.* [4, 5] have shown that observational entropy with appropriate coarse-grainings (in energy and particle number) reproduces equilibrium thermodynamic entropy for canonical and microcanonical ensembles, and provides a definition of non-equilibrium thermodynamic entropy that increases during thermalization.

The quantum Zeno interrogation represents a very different application: a single-particle, pure-state system undergoing repeated projective measurements. Yet the same mathematical structure applies. The coarse-graining in the present case is not in energy but in measurement outcomes, and the "thermalization" is replaced by the refinement of the observer's knowledge through repeated Zeno cycles.

This suggests a deep unity between thermodynamic and information-theoretic notions of entropy, both arising from the observer's limited access to the full microstate of a system.

6 The Lossless Limit and Perfect Knowledge

6.1 Approaching Zero Entropy

The limit $N \to \infty$ of the lossless system represents an idealization where:

$$\eta \to 1, \quad S_{obs} \to 0, \quad P_{abs} \to 0. \tag{22}$$

In this limit, the observer gains certainty about the object's presence without any photon being absorbed—a form of "interaction-free" measurement that is genuinely interaction-free.

The approach to zero entropy follows Eq. (21):

$$S_{obs} \sim \frac{\pi^2 \ln N}{2N} \to 0. \tag{23}$$

For practical purposes, achieving $S_{obs} < 0.1$ (in nats) requires $N \gtrsim 50$, and $S_{obs} < 0.01$ requires $N \gtrsim 500$.

6.2 Physical Limitations

Of course, no real experiment achieves the lossless limit. Kwiat *et al.* [1] observed that optical losses fundamentally constrain the achievable efficiency. A photon contributing to P_{QI} must remain in the system for all N cycles, accumulating losses, while a photon contributing to P_{abs} may be absorbed early and escape further losses. This asymmetry causes η to reach a maximum at finite N before declining.

From an observational entropy perspective, losses introduce irreducible uncertainty: some photons are lost to neither "QI" nor "abs" outcomes, and the normalization of probabilities must account for this. The entropy of the measurement no longer approaches zero but saturates at a finite value determined by the loss parameters.

Nevertheless, the conceptual point stands: in the idealized limit, the quantum Zeno effect enables the extraction of complete information about an object's presence without any physical interaction in the sense of energy or momentum transfer.

7 Discussion

7.1 Summary of Results

This analysis has demonstrated:

1. The observational entropy framework of Šafránek, Aguirre, and Deutsch provides a natural language for analyzing the information content of quantum Zeno interrogation measurements.
2. Observational entropy S_{obs} decreases monotonically with increasing efficiency η (for $N \gtrsim 5$), approaching zero in the lossless limit as $N \to \infty$.
3. The decomposition $S_{obs} = S_{Sh} + \langle S_B \rangle$ separates the uncertainty about which outcome occurs (Shannon) from the uncertainty about the microstate within each macrostate (Boltzmann).
4. The approach $S_{obs} \to 0$ corresponds to perfect knowledge about the object's presence without absorption—a striking demonstration of the relational nature of quantum information.

7.2 Broader Implications

The quantum Zeno interrogation experiment, viewed through the lens of observational entropy, exemplifies several deep features of quantum mechanics:

Relationality: The entropy S_{obs} is not an intrinsic property of the system but depends on the observer's measurement capabilities. Different coarse-grainings yield different entropies for the same quantum state.

Information without interaction: The limit $\eta \to 1$ shows that information transfer does not require energy transfer. The coupling between photon and object must exist (enabling the possibility of absorption), but the *actual* transfer of energy can be made arbitrarily improbable.

The role of possibility: The Zeno effect works because each cycle could have resulted in absorption. The sequence of non-absorptions is informative precisely because absorptions were possible. This resonates with the consistent histories interpretation [9, 10] and with relational approaches to quantum mechanics [6].

7.3 Open Questions

Several questions remain for future investigation:

1. How does the observational entropy analysis extend to quantum objects in superposition states, where the interrogation creates entanglement between photon and object [1, 11]?
2. Can the framework be applied to the resonance-based interaction-free measurements using high-finesse cavities [12, 13], and how do the entropy behaviors compare?
3. What are the implications for quantum thermodynamics when the "system" is an object being interrogated and the "bath" is the stream of interrogating photons?

8 Conclusion

The quantum Zeno interrogation experiment of Kwiat *et al.* provides a vivid illustration of how information can be extracted from a quantum system through carefully designed measurements. The observational entropy framework offers the right mathematical tools to quantify this information extraction, revealing that the observer's uncertainty decreases to zero in the lossless limit of infinitely many Zeno cycles.

This analysis reinforces a relational view of quantum information: what matters is not some absolute property of quantum states, but the relationship between observers and systems as mediated by measurements. The entropy S_{obs} captures this relationship precisely, unifying thermodynamic and information-theoretic perspectives in a framework that applies equally to thermal equilibrium and to single-photon quantum interrogation.

The fact that complete information about an object's presence can be obtained without any photon absorption remains one of the most counterintuitive and beautiful predictions of quantum mechanics, now confirmed experimentally. The observational entropy framework provides the language to understand exactly what "complete information" means in this context.

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